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Capstone project : Zometool : from 0 to the 4th dimension

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Capstone Project

Zometool: From 0 to the 4th Dimension

LS 400

Michael J. Read

Dr. Scott Waltz

Fall 2010

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Back Story

Like a many students of my generation, I have struggled with mathematics in all of its forms. Even in elementary school, I found the simple procedures of arithmetic difficult to master. The “carrying” and “borrowing” of addition and subtraction were things that overwhelmed me and long division was a mystery that confused me. I have seen the same type of frustration and difficulty in the children with whom I have engaged in service learning. The algorithms which I was exposed to in elementary school did not work for me.

The “intelligence tests” which the school system subjected my peers and me to were a bane and a terror. I did well on the English grammar sections and could hold my own on every part of the test except the math. I particularly remember struggling with spatial relationship tests where I had to look at a folded out mathematical net and decide what kind of solid that net formed. I had no spatial relationship ‘intelligence’ and was frustrated that I could not do what others could do.

As a teacher to be though, I knew that I was going to have to grapple with my own “math trauma” and overcome it. I started to investigate different ways of approaching arithmetic. I started to try and invent new ways of doing addition, subtraction, multiplication and addition. Through online research, I became exposed to Vedic mathematics, a system of mental math whose procedures made more sense to me than the ones that I had grown up with.

I have been fortunate to have some really good math teachers at CSUMB who have helped me overcome my own math trauma and to encourage me to “organically organize” the language of mathematics and in the process to inculcate a desire to learn more about that language.

Although there is much research to show that children invent their own algorithms up to the fourth grade, my service learning experience has shown me that children are still struggling with arithmetic and that an expanding of their “arithmetic toolbox” needs to take place. As a teacher, I will do that and really encourage children to think about the base ten number system and how to really internalize it and understand it. Because they are unable to have an organic understanding of mathematics, the increasing of their math skills are put at risk.

The California Core standards for math delineate two areas of concern: number sense and spatial relationships which include geometry and measurement (California State Department of Education, 2010). My interest for the past few years has been on developing a sense of mathematics as a language to be embraced and integrated into children’s lives. I have identified several problems with math education today which I hope to research and develop solutions to during the course of my teaching career as well as to partially address through my project. These problems are delineated by two areas: number sense and spatial relationships. This project will discuss possible solutions to problems in both the teaching and the learning of mathematics.

I have recently been working with a geometric manipulative called Zometool sold at zometool.com. Based on the Fibonacci sequence and the Golden Ratio, this tool allows children to construct the Platonic, Archimedean, and Kepler solids as well as to be creative in their

application of mathematical knowledge. Many other non- spherical constructions are possible with this tool as well, allowing the student to self- scaffold and ask “what if?” questions on his or her own. Learning to work with this manipulative has been revelatory. I am learning to see objects in space in a whole new way. It is my hope to do a workshop with this manipulative at one of the local elementary schools to see if I can increase the geometric and spatial relationships abilities in the students with whom I will be working.

The Nature of the Problem

The overarching problem in math education seems to be the inability of children to internalize mathematical concepts and apply them consistently and with confidence. I have identified several components to this problem which I hope to investigate through my project. First, there is the problem of how to make the link between biologically primary abilities (what we are born with) and those which society demands we learn through education (biologically secondary abilities). Many students do not see the need to learn higher level math and are not motivated to learn anything beyond what is required of them in school.

Secondly, there is the controversy of concept versus procedure in math education. Due to the application of Piaget’s theories in the elementary classroom, there has been a real shying away from teaching children algorithms and instead allowing them to develop means of solving problems on their own. In my experience with service learning however, some children are not learning basic math skills as they should and there needs to be a balance between self constructed procedures and those which have been socially constructed and developed over time (i.e.: the standard algorithms for arithmetic).

In conjunction with the above, math educators need to look critically at the socially constructed arithmetic procedures which are being taught in the textbooks and investigate other cultures’ approach to math education. We have taught the same basic approach to addition, subtraction, multiplication, and division for years and this approach is still causing problems. Thirdly is the question of the manipulative in and out of the classroom. Most students who go into math education have become familiar with base ten blocks, algebra tiles, Cuisinare rods etc. The problem with these manipulatives is that they are for the most part, teacher centered tools.

Research shows (which I will delineate later in this paper) that manipulatives are either used for the reinforcement of material being taught or to increase the retention of that same material. Any scaffolding occurs in the classroom and once the lesson is over, the use of the manipulative ends.

Some of the material presented here will deal with number sense. I have a passionate interest in teaching children to really be competent with arithmetic. To that end I will be discussing our base ten number system and alternative algorithms for teaching arithmetic. For the purpose of my project, however, I will be concentrating on spatial relationships and spatial structuring as they are manifested in geometry.

I have recently been working with a geometric manipulative called Zometool sold at zometool.com. Based on the Fibonacci sequence and the Golden Ratio, this tool allows children to construct the Platonic, Archimedean, and Kepler solids. Many other non-spherical constructions are possible with this tool as well, allowing the student to self-scaffold and ask “what if?” questions on his or her own. It is my hope to do a workshop with this manipulative at one of the local elementary schools to see if I can increase the geometric and spatial relationships abilities in the students with whom I will be working.

I see the workshop which I will be doing with my students as a way to integrate the three principles which I have enumerated above. During the time that I have worked with the students, I have seen a wide variety of a-priori conceptualization. Some of the students instinctively see patterns in geometric shapes and others need encouragement and prodding.

The problem of concept versus procedure is apparent when the students are constructing geometric solids. There are many ways to see for example a icosahedron. One can see it as simply equilateral triangles, a pentagonal anti-prism with two five star rays on either end, or six pentagons interwoven at 60 degrees.

Concept and procedure need to be woven together to really understand the nature and genetics of a polyhedron. You have to be able to physically build the solid but you also need to understand the composite structure as well: How to build it in various ways. Zometool is a self-scaffolding manipulative. When the children build polyhedron with this manipulative, they are encouraged to ask what if questions and to see how they can add on to what has already been constructed.

Literature Review

Linking the child's natural abilities with those that society requires

Elizabeth Carruthers and Maulfry Worthington are two mathematicians from England. For the past few years they have been investigating how children manifest their natural mathematical sense in drawings and “scribbling” as well as how children often find it difficult to make the transition from using their own symbols (manifested in their drawings) to the conventional symbols which are used throughout the world (+, -, \div , = ,etc.) They have published many of the children’s drawings on their website: (Carruthers & Worthington, 2010). In these drawings one can see an amazing variety of mathematical concepts being expressed at an early age. Concepts of many, partitive division, numbers as labels etc. are all represented in drawings from the age of 3.

There are several things which I like about this site. First of all is the range of drawings which illustrate Carruthers and Worthington’s theories about mathematical intelligence in children. Also they constantly update their research into children’s mathematical graphics as well as give honor to those who have gone before in the field of human development.

Both Carruthers and Worthington have co-authored a book entitled *Children’s Mathematics: Making Marks, Making Meaning*. Throughout this book Carruthers and Worthington imply that children want to be able to see the significance of what they are experiencing in the world (Carruthers and Worthington, 2006.) They also imply that teachers should engage the student and parents together to evaluate the child’s development and cognition as they relate to mathematical skill. They do this by comparing the schools of behaviorism (Thorndike and Skinner), constructivism, social constructivism (Vygotsky), and social culturalism (Vygotsky and Bakhtin), (Carruthers & Worthington, 2006, p. 21). Although they give credence to each of these schools of thought, they actively embrace social culturalism and social constructivism.

The authors of this book refer to Wenger’s 1988 study which indicates that when children share their experiences of mathematical concepts through drawings that they are able to better internalize, reinvent and co-construct mathematical language (Wenger as cited in Worthington and Carruthers, 2006, p. 23). I believe that I will be able to demonstrate a similar process through my Zometool workshop. I am planning on building geometrically analyzable constructions through group work and reciprocal teaching. In the process of this I am going to encourage the children to share their ideas with each other. I am also hoping to involve some of the parents of the students that I will be working with as well.

One of the implications of Carruthers and Worthington’s work is that often the assimilation of new mathematical concepts is hindered by the need of children to learn new symbols for concepts which they already know. I feel that this is true. From my own experience with learning math, the hardest battle is to understand what the new symbols in a math book are trying to communicate. Once I learn what the symbols mean, the internalization process becomes much easier.

Although I will be focusing on geometrical spatialization skills, there will be some new language and symbols for the children to learn in the workshop which I will be giving. I plan to break this new ‘code’ down into components which the students can assimilate.

David Geary is a cognitive specialist who is currently working at the University of

Missouri-Columbia. David focuses his research on mathematical learning and evolution. I was intrigued with reading of his description of “biologically primary abilities” and “biologically secondary abilities”. He defines the former as those which are pan-cultural and the latter which may or may not be (Geary, 1995, p.25).

For example human language is found throughout the world but not all people read (Geary, 1995, p. 25). This concept impacted me as being applicable to math education in general. We all soon develop the ability to recognize one to one relationships in counting irrespective of the order in which the objects are being counted but not all of us have facility with the ability to use the symbols of mathematics in a meaningful way. In my use of Zometool, I will not only teach the children to build by connecting a particular strut to a connector/s but also encourage them to think mathematically about what they have built.

Another example which Geary gives is the ability of animals to navigate around their habitat. This ability however, does not imply the biologically secondary ability of being able to read a map and plan a route. The Euclidean knowledge of instinctively knowing that the shortest distance between two points is a straight line is epitomized in the phrase “as a crow flies” (Geary, 1995, p. 26). We take ownership of that fact soon in our lives but more advanced concepts such as congruency of polygons and the prime factorization of numbers take time and education.

In a 1991 study, S. Ceci implies that biologically secondary abilities are acquired through schooling (Ceci, 1991). Ceci strongly believes that an adult’s cognitive processes are informed throughout school. Although Ceci concentrates much of his attention on the efficacy of intelligence tests, his research supports Geary’s emphasis on the importance of education. As many educators know though, there is a “Hidden Curriculum of Work” in which not all students have access to the quality of education which the rich and the affluent have (Anyon, J. 1983). I hope as a teacher to be constantly trying to uncover occult genius in my students regardless of ethnicity or economic background and to work with the knowledge that they do have in order to better link them to the knowledge which is required both by state mandate and the society in which they live.

Geary states emphatically that whereas biologically primary abilities are pan-cultural and based on the interests of the child, the more demanding biologically secondary abilities are mandated by the needs of the society in which the child is immersed (Geary, 2005, p. 28). Children will play video games for hours but will not, unless highly motivated, sit down and work on a math problem. There is no connection between what they want to do and what they are asked to do by their teachers who are trying to prepare their students for a productive life in the society to which they belong.

I believe that through working with Zometool, I can initiate real mathematical inquiry in the children with whom I will be working with. Although the United States has improved its rankings in the International Mathematics Olympics, it is apparent that this country is really behind China which has ranked first for several years (International Mathematics Olympiad, 2010). Therefore, there is an apparent need to really address mathematical competence in this country.

I have been speaking about the need to link primary math abilities with those that are secondary. Annie Han is an assistant professor of Mathematics at the City University of New York. During a trip to China, Han noticed that first grade mathematics classes are actually taught by teachers who had specialized in mathematics (Han, A., 1999). My service learning experiences with elementary school teachers is that they often teach out of the book without

really understanding mathematical concepts. I believe that it would be profitable if not convenient to instigate the Chinese method in this country in order to insure the mathematical competency of all students through the use of mathematically qualified teachers.

There is much concern amongst math teachers on how to best teach mathematics. It is generally agreed that those secondary abilities involving math skills depend on a lot experience and exposure to concepts. A child's natural math abilities will be "fleshed out with the child's natural abilities" (Geary, 1995, p. 32). These natural math abilities are the framework or skeleton upon which the more complex abilities are grafted (Gelman, 1990). Perhaps we can use the analogy of a function which has an input and an output. We take the child's natural abilities and put them through the 'function machine' of education and we get an output which hopefully has shape and direction. There are some functions which require the output from other functions though. The output from one function becomes the input for another.

The analogy I am making is that from the beginning of a child's education there needs to be a cohesive relationship and agreement between all three components of the 'function machine': The teacher, student, and the curriculum. If the output at a particular stage of a child's level is faulty (lack of competence in an academic discipline) then what is inputted into the next level 'function machine' will produce a faulty output and the cycle continues till the student at some point gives up and fails.

By properly linking the student's natural abilities to higher level thinking (secondary abilities), a fruit-bearing experience will occur between both student and teacher. I hope to investigate some of these ideas in my Zometool workshop. I will constantly try to be aware of my students' natural abilities while increasing their zone of proximal development through reciprocal teaching and participation, becoming a link in the process.

Concept versus Procedure in Math education

James Hiebert of the University of Delaware and Thomas Carpenter from the University of Wisconsin are concerned with the understanding of mathematics by children. They define understanding in terms of the structure of information and the way it is represented (Carpenter T. & Hiebert J., 1992, p. 65). The structures of information may be thought of as the concepts which need to be learnt in solving problems as well and the way they are represented as an algorithm or procedure for solving a particular problem. Mathematical concepts need to be internalized through a series of connections which make the knowledge organic and meaningful.

In order to understand a process such as two or three column addition, a student has to make connections with various pieces of information. He or she has to understand the 'base ten' number system As well as to know the difference between what has been added and what still needs to be. It seems simple to an adult that only one digit from the running total of a column remains in that column and the other digits from that sum are carried over to the next column but is a process, which for a child, often needs to be wrestled with.

For a child though these simple steps are often very difficult to accomplish. Educators like Constance Kamii discourage the teaching of algorithms or procedures to a child. Instead they are encouraged to come up with their own procedures and methods for solving arithmetic problems. She implies that "logico-mathematical knowledge" is increased by an individual forming his or her own relationships with the material being taught (Kamii, 1989, p.5). I agree with this to an extent. A moth needs to struggle out of his or her cocoon in order to pump blood into its wings to fly. If you release the moth from a cocoon without that struggle, the moth will

die.

That being said, not all children come up with workable procedures by themselves and therefore need guidance. I believe that the perceived dichotomy between the teaching of concept and procedure is a misperception. Hiebert and Carpenter believe that both types of knowledge are needed (Hiebert and Carpenter, 1992). Memorized procedures allow the rapid solution of mathematical problems. Conceptual knowledge fortifies and brings understanding concerning the reason why a particular procedure works. For example, an even number is one which can be divided by two and that an odd number is one that cannot. The underlying concept is that an even number can be written as $2k$ (where k = any constant) and an odd number is one that can be written as $2k+1$. One may recognize an even number by one that ends in 0,2,4,6, or 8. That is an easy to remember procedure. It is justified by the previously mentioned concept. Both are intimately related. The former is part of the internalized network of mathematical connections and the latter allows the student to just draw on the concept without having to labor over the reason why it works.

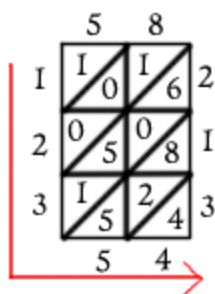
An author who has written extensively on this problem is Jean Schmittau from the University of New York. Schmittau points out that mathematical procedures were developed by human beings who encountered increasingly difficult problems in their daily life which required foolproof and rapid methods to solve them (Schmittau, 2004). As mentioned earlier, concept and procedure are intimately intertwined. One provides the solid foundation for the other. Schmittau also points out that there needs to be flexibility and variability in the solving of mathematical problems. One size shoe does not fit all.

Schmittau mentions the 1977/1992 study by Scribner who described the way dairy workers filled an order. If they were given an order for one case of sixteen bottles minus six, they add two bottles to a partial case of 8 rather than remove 6 from a filled case (Scribner, 1997/1992 as cited in Schmittau, 2005). This is a case of “cognition in the wild” (Schmittau, 2005, p.23). The dairy workers showed the flexibility and variability needed to solve the problem.

Schmittau points out that one’s cognitive development occurs when those methods which have been previously used to solve a problem are inadequate. She is a Vygotskian in that she believes students should be exposed to increasingly scaffolded problems which require the student to adapt new methods to new problems. Like Hiebert and Carpenter, she sees an intimate relationship between procedure and concept (Schmittau, 2004, p. 23).

Much of my mathematical interest over the past few years has been to investigate different algorithms for doing basic arithmetic. The following image shows a commonly used method to teach multiplication and shows the result of 58×23 .

Each box contains a diagonal in which the respective digit is placed after a multiplication, for example: $2 \times 8 = 16$ (the upper right box contains the digits 1 and 6), $2 \times 5 = 10$ (the upper left box contains the digits 1 and 0). In turn each digit of the multiplicand is multiplied by the digits of the multiplier and the result is placed in the box directly below and across from the corresponding digits. The sum of each diagonal is written outside of the box. When completed the student reads the answer from left to right around the exterior of the box: (12354).



<http://en.wikipedia.org/wiki/Shabakh>

This method is one of several different approaches to multiplication. It incorporates the same elements as the FOIL method which most of us were taught in beginning algebra to solve binomial multiplication. The method is effective and should be a part of every elementary school teacher and student's tool box.

After looking at several different ways to approach multiplication, I have adopted vertically *and crosswise* as described by Indian Mathematicians who follow the system known as Vedic Mathematics. Vedic mathematics was developed by an Indian Mathematician Sri Tirthaji who lived from 1884-1960. I have read his book *Vedic Mathematics* and much of what I have learned about Vedic mathematics comes from this publication (Tirjathi, 1992).

The following problem shows the result of 25×37 . In the traditional manner we would multiply 7×5 , write down **5**, carry the 3 adding it to 2×7 , write down **17**, multiply 3×5 , go to the next line, stagger to the left, write down the last digit of **5** from 3×5 , carry the one and add it to $2 \times 3 = 7$, bring down the **5** from the first line of computation, add 7 from the first line to the 5 in the second, write down the **2** from 12, carry the one and add it to the sum of $1 + 8$ to get a final answer of **925**:

The Vedic method for this same problem is, to my mind a lot simpler and requires few steps. We work with place value in the following manner: Write down the sum of all the ones ($7 \times 5 = 35$), then the tens ($(2 \times 7 = 14) + (3 \times 5 = 15) = 29$) and finally the (100's = $2 \times 3 = 6$). We then have one line which reads **06 29 35**. Since we know that each place value holds only one digit in the final answer we can simply write down the answer by working from left to write and carrying

when necessary to get the same answer as above but in my opinion much quicker and with fewer steps. The same method can be employed when doing any length multiplication problem. One simply has to work with place value and group all of the products which refer to a particular place value together.

$$\begin{array}{r}
 25 \\
 37 \\
 \hline
 06 \ 29 \ 35 \\
 \hline
 9 \ 2 \ 5
 \end{array}$$

The image shows a handwritten multiplication of 25 by 37. The numbers are written vertically. A horizontal line is drawn under the second number, 37. Below this line, the products are written in three groups: 06, 29, and 35. Another horizontal line is drawn under these products. The final result, 925, is written below the second line.

Tirthaji offers an interesting application of “vertically and crosswise” by showing how the memorizing of one’s times tables above 5 is not necessary. If we are multiplying two single digit numbers together like **9x7** we get the answer in the following manner : 1: write down the problem in the traditional manner and then place a hyphen next to both numbers. To the right of the hyphen, write down the deficiency from ten for both numbers (9-1: 7-3) .The second digit is $3 \times 1 = 3$ **and** the first digit is the difference of each deficiency from the other number being multiplied ($9-3 = 6$, $7-1 = 6$). The answer therefore is **63** (Tirjathi, 1992, p. 12). As one can it does not matter whether one subtracts 3 from 9 or 1 from 7, the left most digit is **6**. This method is justified algebraically by $(x-a)(x-b) = x(x-a-b) + ab$. By way of explanation: $(10-1)(10-3) = 100 - 10 - 30 + 3 = 10(10 - 3 - 1) = \mathbf{63}$. Of course if we have determined the digit on the right and there is an excess (greater than nine), we carry it to the left.

Tony Barnard from Kings College in London and David Tall from Warwick University in Coventry, England, have done some considerable work with what they call “cognitive units” (Barnard and Tall, 1997, p. 41). They define this unit as “a piece of cognitive structure that can be held in the focus of attention all at one time” (Barnard and Tall, 1997, p.41). In the two different procedures for doing multiplication, the latter seems to be the one in which a cognitive unit is easily discerned. In the above example there are only three cognitive units: The ones, the tens, and the hundreds. In the case of the tens there are two smaller units (1’s x 10’s & 10’s x 1’s).

One of the major problems which math students at any level may encounter is the ability to take the information which they are given and fit it into cognitive units which make sense to him or her. This goes back to the nature of language which is deriving meaning from symbols.

By offering the student several procedures backed by sound concepts, computation will become a lot more facile. Mathematics is a lot like reading. In reading one learns that C-A-T spells cat. The individual cognitive units have been internalized and combined into a new cognitive unit, “cat”.

Of course in the cases of the spoken and written languages, there is an ongoing social reinforcement of the relationship between the symbols used for the constructs of that language and the meaning which is derived from those constructs. The problem with the teaching of mathematics is that the symbols used (especially in advanced mathematics) to signify concepts often do not evoke a sensual response in the student.

For example if I read “Out of the night that covers me, black as the pit from pole to pole, I thank whatever gods may be for my unconquerable soul...” the derived meaning at some level is easily internalized and understood. Many of the cognitive units (individual words) have been seen before. If on the other hand a math student sees $(m \in n) \cup (S \in x)$ or $\int x^3 + \ln x \, dx$, he or she is often mystified because none of the symbols are familiar or have sensual reference points to derive meaning.

The problem of subtraction

Most elementary school teachers will tell you that their students find subtraction difficult. The “carrying” and “borrowing” is frustrating to the children and confounds their attempts to get a correct answer. For the past few years I have been practicing alternative ways of subtraction to see which made the most sense to me. Steve Wilson from Sonoma State University has a web page in which he offers alternative methods of subtraction (Wilson, 2004).

Of particular interest to me is the Austrian method which he mentions on his website and is the focus of a 1997 study by Carla Fiori and Luciana Zuccheri. The Austrian method is illustrated below next to the standard American procedure:

$ \begin{array}{r} 15 \\ 765 \\ - 39 \\ \hline 726 \end{array} $ <p style="font-size: small; color: blue;">I am adding a 10</p> <p style="font-size: small; color: blue;">I'm adding one here too.</p> <p style="font-size: small;">Borrow and Pay Back Method Or Equal Additions</p>	$ \begin{array}{r} 515 \\ 765 \\ - 39 \\ \hline 726 \end{array} $ <p style="font-size: small;">Borrow and Regroup Method Or Decomposition</p>
---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------

<http://www.littlehouseinthevalley.com/wp-content/themes/images/subtraction.jpg>

As one can see, in the Austrian method the subtrahend is increased by one rather than diminishing the minuend like the example on the right. Fiori and Zuccheri showed that students using the Austrian method are less likely to make mistakes than those who use the traditional method (Fiori and Zuccheri, 2005). If one really thinks about it; the “borrow and pay back method” (Austrian method), really gives a more accurate picture of what is happening in subtraction.

In the above example, I am decomposing one ten from the second column to give ten ones to the first column. I am in fact, therefore, increasing the amount subtracted from 6. I change the 3 which is subtracted from 6 to 4 subtracted from six and then proceed.

Although procedure and concept are intimately intertwined, concept must be enforced over procedure. Whatever method a student uses, he or she must have a firm understanding of why the procedure which they are using works. I have found that the Vedic method illustrated below makes sense and when understood, helps to internalize the often difficult to understand procedure of subtraction.

$$\begin{array}{r}
 8,280,265 \\
 -2,730,893 \\
 \hline
 6\underline{5}, 50\underline{6}, 3\underline{2} \\
 \hline
 5,549,372
 \end{array}$$

In the illustration above, the problem of $8,280,265 - 2,730,893 = 5,549,372$ is illustrated. Immediately below the minuend and subtrahend, I have written each column's sum. The negative sums are underlined (i.e. $6-9 = -3$). The rules for Vedic subtraction and the other basic operations of arithmetic are explained by the Vedic Math Academy (Vedic Math Academy, 2010). Below is a summation of the procedure of subtraction using the "first from 10 and the rest from 9" rule:

1: if a column sum is positive write it down ($5-3 = 2$)

2: When entering a complement system (**negative sums**), determine the absolute difference between the subtrahend and minuend from 10 ($6-9 = -3$): Subtract the absolute difference from 10: ($10-3 = 7$).

3: While still in the complement system subtract the next difference from 9 ($9-6 = 3$)

4: $0-0 = 0$ is a special case while in a complement system. Since there is "borrowing" from the the current column to the one on the right, this sum is actually -1 and therefore still in the complement system. As such we have ($9-0 = 9$) and rules one or three apply for the next column to the left.

5: When exiting a complement system, simply reduce the positive sum by one ($5-1 = 4$).

For the last two columns moving from right to left we enact rule 2 and 5 ($10-5 = 5$ and $6-1 = 5$)

The answer is from left to right then: **5,549,372**. All five rules can be encapsulated in the Vedic sutra (rule) of “the first from 10 and the rest from 9”. (Tirjathi, B.,1992.p.12). It is interesting to note that I actually came up with this method in 2006 on my own while trying to “invent” new ways of doing arithmetic. My codification of the technique was a lot more complex than this simple procedure and consequently, I have adopted and internalized the above mentioned procedure and use it exclusively. I hope at some time to test these procedures on my students.

In a recent conversation with Dr. Scott Waltz of CSUMB, I pondered why teachers teach procedurally rather than conceptually (Read, 2010.) It was an interesting talk. We came to an accord that perhaps they teach the mathematical procedures that they do because those which are applied in the classroom are those which have been time tested to work. The problem though, is that an inordinate amount of errors are still being made by the students.

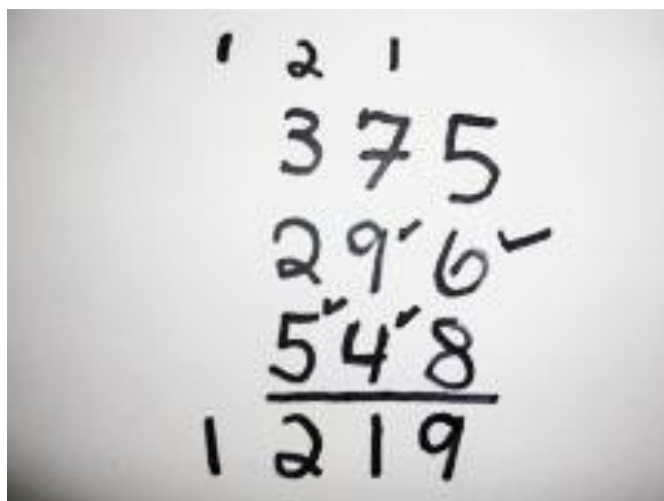
The procedures which we do use in the classroom are not as effective as the teachers who use them claim. Students are still making conceptual errors while using those procedures. There are many different approaches to teaching arithmetic and perhaps there is no one “correct method” which works in all cases. The solution is to increase the student’s “toolbox” so that they have a wide variety of procedures to follow. In addition, this would make math education more interesting for the students.

It is not my purpose in this project to illustrate and discuss every alternative algorithm for arithmetic but to emphasize that continuing research needs to be done as to effectively help children internalize and apply mathematical procedures. More dialogue needs to occur not only between teachers but also between students and teachers as to what works and what does not. Indian mathematics has undergone a real resurgence in the past few years. Vedic and Abri math have gained exposure through sites like youtube.com. The resources are out there for both teachers and students to delve into and explore in order to help students internalize the concepts and procedures which they need to use to do basic and advanced mathematics.

Vedic Addition

Vedic addition is similar to the pattern which we use in the West. Columns of numbers are added and the surplus is carried over into the next column. The difference is that in Vedic addition, one never counts higher than nine. When one gets a number higher than nine, a tick is placed next to that number and just the digit which belongs to that place value is kept to start a running total. At the end of the column, the running total is written down and the ticks are added up. These sums of these ticks are then carried over into the next column and the process starts again. This method is similar to the Trachtenberg system but in my opinion, easier to execute as in this method one counts up to 11 rather than 9 (Cutler and McShane, 1960, p.105-131). The addition below shows how the Vedic system is used. One never counts higher than nine. A tick is placed next to the number which is greater than nine. The difference between the number and ten is then the start of a new running total. The ticks are then ‘carried’ over to the column to the left and become the start of a new column sum.

I have found this procedure to be very fast and efficient. There is no reason to carry more than digit at a time in one’s head as there is only one digit per place value in the final answer.



Both the pictures and the descriptive text which I use to explain Vedic math algorithms are taken from an extensive research paper on Vedic math and the culture of India which I did as a joint paper for my discrete math class with Dr. Lipika Dea as well as the class on global education with Dr. Paoze Thao. I have been thinking about the problem statement for this project for several years.

A workable division algorithm

To be complete in discussing alternative arithmetic algorithms, I offer the following illustration of the technique known as double division. I really like this algorithm because it takes the guessing out of long division. It does take a little longer but there is no trial and error guessing when determining the best partial quotient. Division next to subtraction seems to be the most difficult procedure for children to understand.

Division is usually taught as a “gazinta” concept: How many times does the divisor ‘gazinta’ the quotient? Students do not really understand that though. Division just becomes another procedure which is performed without any real concept of what is happening. The technique of double division makes sense, not only because it takes the guess work out of deciding the best multiple to use for the quotient but also because it uses whole numbers rather than individual digits.

DOUBLE DIVISION WITH 3 DIGIT DIVISORS (Longer but no trial and error)

First double the
divisor 3 times.

Then use the best
multiple here.

1x = 357	768949	
2x = 714	714000	= 2000 x 357
4x = 1428	54949	
8x = 2856	35700	= 100 x 357
	19249	
	14280	= 40 x 357
	4969	
	3570	= 10 x 357
	1399	
	714	= 2 x 357
	685	
	357	= 1 x 357
	328	
	2153	answer
		remainder 328

In summary, children come into the school system hardwired to be able to distill meaning from concepts. If they are given encouragement and direction they will learn what they are supposed to at every step in their education. Effective and flexible procedures must be linked with concepts to insure that student's are able to both internalize and apply the knowledge which they learn in the classroom.

Zometool as a geometric manipulative

Most children going through school have been exposed to the uses of manipulatives in the classroom. Algebra tiles, base ten blocks, cuisinaire rods etc. are all tools to help both the teacher to teach and the student to learn in the math classroom. In effect though, the manipulatives which are used in mainstream schooling are use either to reinforce what the teacher is teaching or to increase material retention during a lesson being taught (Canny, M., 1963). Once the lesson is over, the use of the manipulative ceases.

The beauty of Zometool as a manipulative is that it is a 'self-scaffolding manipulative' which the student can use in the classroom and at home to increase his or sense of spatial relationships and geometry. Although my interests as a math teacher encompasses both number sense and spatial intelligence, my project this semester focuses on the latter.

The importance of geometry was emphasized by Albert Einstein at a conference with the Prussian Academy of Science in 1921. Einstein emphasizes that geometry is the vehicle used for understanding real objects (Einstein, 1921). Geometry in its root meaning implies 'measurement of the earth and all that is in it'. In the same conference report, Einstein says that his work on relativity would not have been possible without the tenets of Euclidean Geometry. He also says that axiomatic geometry (governed by rules) is not enough to really understand reality. Those rules must be coordinated with experience (Einstein, 1921).

Zometool gives the user hands on experience to coordinate both theories and axioms with reality. As stated earlier, the Core Math Standards of California state the need for spatial intelligence and geometry (Department of Education, 2005).

Since I knew that I was going to be working with Zometool, I knew that I need to come with a workable definition of the ability to see things in n dimensional space and extract meaning from them. Several math education researchers from both Kent State University and the State University of Buffalo, New York have said "We define *spatial Structuring* as the mental operation of constructing an organization of form for an object or set of objects. Spatially structuring and object determines its nature or shape by indentifying its spatial components, combining components in spatial composites, and establishing interrelationships between and among components and composites." (Arnoff, Battista, Clements et al., 1998, p. 502-503).

The authors then go on to say that this process requires the student to select, coordinate, unify and store the information in his or her mind (Arnoff, Battista, Clements et al, 2005, p.3) This definition and comments seem to me a good way to focus what I want to accomplish with my workshop.

Rebecca Ambrose is a math professor at University of California at Davis. She and her colleague Garrett Kenehan from the Berkley College of Music have been concerned for some time with the componential reasoning of math students which involves the ability of perceiving

an object as a whole and breaking it down into several components. They both agreed with Battista that children would progress from looking at the whole to the relationship of the parts as they gained experience from working with polyhedra (Battista as cited in Ambrose and Kenenhan, 2009, p. 160). They also say that image based reasoning is not sufficient in order to develop spatial reasoning. They believe that the hands on building of polyhedra should promote conflicts in student's reasoning so that they go beyond the mere visual perception to the actual componential structure of the individual polyhedral (Ambrose & Kenenhan, 2009, p.160).

To prepare myself for this workshop, I have not only worked with Zometool but also immersed myself in the study of polyhedra and the concepts of symmetry. George Hart is a geometric sculptor who has done a lot of work both with Zometool and also in analyzing the structure of polyhedra (Hart, 2000). I have studied both his writing on polyhedra and his Zometool constructions. Herman Weyl says that "*Beauty is bound up with symmetry*" (Weyl, 1952, p. 3). One of the things which I will emphasize with my Zometool workshop is the ability of students to visualize the axes of symmetry for any geometric solid. For example the cube has axes of symmetry around the midpoints of opposite faces as well as along the diagonals. For complex solids, detecting symmetry is often a difficult task for the student and one which I will practice in my workshop.

The front page of George Hart's *Zome Geometry*, offers the following thoughts from Galileo: "Truly I begin to understand that although Logic is an excellent instrument to govern our reasoning, it does not compare with the sharpness of geometry in awakening the mind to discovery." (Galileo as quoted in Hart and Picciotto, 2001, p. ix). This summarizes what I hope to do with my students. Although I do plan to engage in logical dialogue with them, I want my workshop to be one of discovery and joy. I will be relying a great deal on Hart and Picciotto's text in that solid and plane geometry is explained using the structures of Zometool. Although some of Hart and Picciotto's text is not at this time applicable to elementary school children due to its mathematical complexity, there is much to be gleaned from the probing questions concerning spatial relationships which are presented in their book.

Besides Herman Weyl's book on the components of symmetry, and the geometry textbook of Hart and Picciotto, I have found that there is much to ponder in H.M. Coxeter's book *Regular Polytopes* (Coxeter, H., 1963). This book is very helpful for defining and analyzing the polyhedra. For example the polyhedron are described as "a finite, connected set of plane polygons, such that every side of each polygon also belongs to just one other polygon, with the proviso that the polygons surrounding each vertex form a single circuit..." (Coxeter, H., 1973, p.4).

Most of us can recognize a cube when we see it but most elementary school children would be hard pressed to define its characteristics by the above given definition of a regular polyhedron. It is my intent to encourage my students to think precisely about geometric shapes. For example the cube is comprised of 6 faces, 3 each of which meet at each vertex and there is only one circuit around each vertex of this solid. This book was recommended to me by David Richter of Western Michigan University. David is a brilliant geometer who has done a lot of work with Zometool (Richter, 2010).

Rebecca Ambrose of the University of California has also communicated with me both my email and via telephone on my projected workshop with Zometool. Both Rebecca Ambrose from UCD and Garrett Kenenhan from the Berkley College of music have been concerned with the mathematical performance of our youth in respect to geometry (Ambrose R. & Keenan, G., 2009).

I spoke at length with Rebecca about her work with children and wanted to get some feedback about Zometool. Rebecca prefers to work with faced based geometric constructions like Polydrons since the learning curve for this product is much shorter. I agreed with her to a point but pointed out that once the basics of Zometool were acquired, the complexity of objects which can be built with Zometool is much higher than with faced based tools such as Polydrons. For clarification, Polydrons uses already constructed planar forms while Zometool uses struts which must be assembled into the required shape. I conceded that Polydrons would perhaps be a better way to go in the beginning with very young children but that Zometool offers many more possibilities for constructing complex shapes. It is my intention to use not only Paul Hildebrandt from Zometool but also math educators from the colleges that I have contacted as resources. In this respect, I can call them community partners in my project.

Summary of literature review and final comments

Like anyone who is doing mathematical education research, I have realized that the subject is broad and multifaceted. I have discussed several facets of math as a language which are the linking of the young student's natural abilities with those that he or she must learn in school, the ongoing problem of whether to teach children, the standard procedures of arithmetic or to intensely investigate different algorithms for doing the same, and finally to discuss Zometool as a effective geometric manipulative to teach spatial relations in three dimensional shapes.

Like any other language, mathematics can be simple or complicated depending on the level of communication required. There is a great deal of literature about the cognitive processes involved in learning even basic arithmetic. I have just scratched the surface but plan to research that literature in order to both inform myself and to hone my craft as a future teacher.

In addition, I believe that the key to economic access in this country specifically and the world at large is to become proficient in mathematics. There is a hidden culture behind the computers of our technological world and that culture is mathematics. We no longer live in an age where it is just sufficient to speak English (or one's native tongue) and be able to write well. Those skills perhaps gave all the potential for political access but not for economic access and power (Moses, 2009).

Through enabling a student to make the connection between his natural abilities and the requirements of society, an increasing of both procedural and conceptual fluency and the reinforcement, retention and creative development of mathematical procedures through the use of manipulatives, I believe that a student will gradually learn to speak the language of mathematics which is so necessary in this global market economy. In addition, I also believe that there is a beauty in mathematics which transcends either the technical or practical and can enrich both the life of the teacher and the students with whom he or she is working.

Community Partner

The knowledge which I gained from doing my literature review for this project was both informative and edifying. It was informative in that I learned gained new insights into some of the problems surrounding math education and edifying because I saw that there were resources which were both of a physical nature (Zometool) and pedagogical (such as the writings of Geary, Ceci, Schmittau, etc.) which I can avail myself of as a future teacher.

My community partner for my project was Mckinnon Elementary School under the supervision of Susan Fisher, its principal. We both see the need of hands on projects which will increase children's mathematical sense. To that end I worked with six 5th grades that Susan has chosen. Susan and I have been talking about doing a presentation for the local school board as well as for the other teachers at Mckinnon. I am grateful to have a community partner who is also enthusiastic about what I am and will be doing with my students.

Paul Hildebrandt of Zometool is also my community partner for this project. On August 11th I attended a Zometool workshop for math teachers which Paul gave in Clairmont, Ca. I was grateful for the opportunity to learn from others and gain valuable experience for my own forthcoming workshop. We spent the whole day building geometric constructions in small groups and then gathering together to talk about what we had learned in the process. I spoke with one of the geometry teachers who used Zometool in her classroom about my forthcoming workshop and she gave me some very useful advice about having a specific goal for each session with my students.

Paul and I have formed a long distance friendship and talk from time to time via email and on the phone about what I am currently doing with my students. He has been very helpful and encouraging. We both agree that we would like to see Zometool in every child's home.

Project Plans

For the duration of this semester I plan to develop spatial structuring abilities in the students with whom I will be working and to increase my students' number sense in the process. I will be doing the workshop for two days a week. On Tuesdays I will be meeting with my students from 7:40 a.m. to 9:00 a.m. On Thursday, I will be giving the workshop from 2:40 p.m. to 4:00 p.m. It may occur that I will have to expand this schedule to one other morning if the need arises. The components of the Zometool construction kit are illustrated as follows:



This is the Zometool connector ball which has three differently shaped holes for each type of strut. Each strut's end is shaped to fit into one type of hole. The blue strut fits into the rectangular hole and represents the sides of a square or rectangle. The yellow strut fits into the triangular hole and it represents the height of an equilateral triangle. The red strut fits into the pentagonal shaped hole and represents the height of a regular pentagon (5 sides of equal length). The pincushions formed by inserting each different

color of strut in to the connector ball are illustrated here:



The blue pincushion has 30 struts, the yellow has 20 struts and the red has 12. This gives a total of 60 different directions. At one time, these were the only struts available for Zometool but in the late 1960's, artist Jean Baudoin and designer Fabien Vienne collaborated to come up with a new green strut with an angle on both ends.

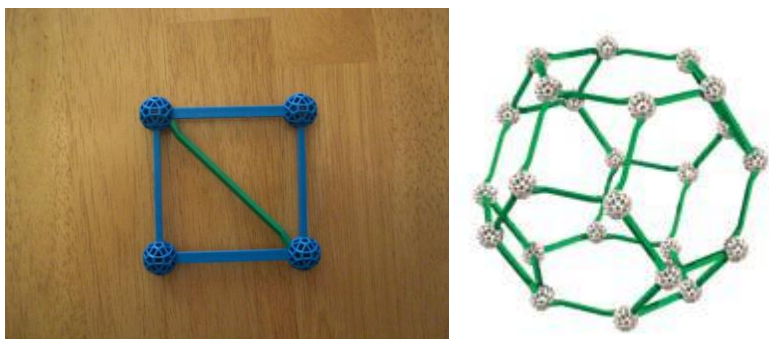
into the



This strut which fit the hexagonal hole of Zometool connector, inserted into the hole tetrahedron (4 faced

allowed 5 new directions based on the way the strut was as seen in this picture of the vertex pattern of a solid consisting of equilateral triangles) inscribed in a cube. This innovation greatly increased the number of constructions which were possible with Zometool.

Its length as seen in the picture below is the hypotenuse of a right triangle whose sides are 1 and $1 = \sqrt{2}$. The truncated Octahedron to the right is only possible using the green struts.



Besides teaching the children to construct geometric objects with Zometool, I plan to familiarize them with several mathematical concepts related to geometry. Topics like the Fibonacci sequence, Golden ratio, Euler's formula which shows a relationship between the number of faces, vertices and edges, and the Pythagorean Theorem will be discussed on Tuesday mornings. We will also be doing compass and straight edge constructions as well.

In the short time that I have been working with these students, I have been able to group them in terms of possible expertise. Two of the students will be concentrating on the zonohedra (polyhedra consisting of parallelograms meeting at a vertex and bands of parallel lines going around the solid) two of the others will become experts on the Platonic solids and the remaining two will concentrate on the Archimedean solids such as the truncated cube and truncated icosahedrons.

The main building project will be the construction of a 3d projection of a 4d hyper-dodecahedron known as the 120 cell because it will consist of 120 dodecahedrons which are progressively flattened as they move from the core of the structure to its surface. This will be an exciting project because I will try to get the students to conceive of a 4th dimension beyond height, width and length. We will be constructing this soon and discussing it as the semester progresses.

It is also my intention to do some compass, protractor, and straight edge work with the students as this activity is part of the state standards for their age group. We will be learning how to bisect both an angle and a straight line, erect a perpendicular to a point on a line as well as inscribe polygons inside of a circle.

Each day I will be assessing the student's knowledge through the KWL form and daily journals. It is my hope that the 6 students with whom I will be working with will come away from this experience enriched with more mathematical knowledge than they had before and encouraged to pursue geometry on their own.

For the deliverables of my project I will be including the actual workshop journals of my work with the students, and two quizzes that I have designed. Next semester I will be building on the assessment section of my Zometool workshops.

Significance of the Project

As stated before, the California State Core Standards for mathematics state that mathematics education in the early grades should concentrate on both number sense and geometry which includes spatial relationships and measurement (Sacramento Office of Education, 2010). Although math education in k-6 will concentrate on number sense, I see a real need to emphasize physical dimensional geometry in k-6.

The students with whom I have been working with are bright and eager to learn. There are, however, gaps in their education as far as their understanding of the relationships of various shapes to one another and the attendant properties of those shapes both individually as a group and individually. Although Euclidean geometry has fallen out of repute somewhat (Fletcher, T., 1971), I see a real need to maintain it in our elementary schools. There is no doubt that transformational geometry and the study of vectors is appropriate for high school and middle school students but our k-6 students need to progress according to the Van Hiel levels.

The first two of these levels as Ambrose discusses them are the 'visual holistic' and the 'descriptive/analytical' (Clements and Battista, 1972 as cited in Ambrose and Kenenhan, 2009, p. 159). The students with whom I have been working seem to be able to recognize the basic polygons such as the triangle, square, pentagon, hexagon etc. Their analytical and descriptive skills which require the seeing of the relationships between these shapes however, are limited. I see a real need to develop this area of their education.

Simple tasks like just counting the edges on a cube are often difficult for young children and they often count the vertices rather than the edges (Ambrose, R., 1997, p. 1). The problem with 2d representation of 3d objects is that that representation only shows a part of what is being represented. By doing hands on building of polyhedral, children can really see the actual components of a particular solid.

In 1991 Vinner proposed that the image based reasoning of a child is predominant because that type of reasoning is sufficient for many of the problems which a student is given (Vinner, S. 1991 as cited in Ambrose and Kenenhan, 2009, p. 160.) Children need hands on experience with building polyhedra in order to really develop componential thinking in respect to three dimensional objects.

Froebel knew the value of hands on experience with shapes and geometric manipulatives. Scott Bultman points out that the gifts were useful in developing sophistication in geometry and physics but that also a sense of creativity and play can develop through their use (Bultman, 2001-2008). I see my Zometool workshop as achieving the same.

Vinner and Kershkowitz point out that the visual image that a student has to represent a geometrical concept may not be congruent with the formal definition (Vinner and Kershkowitz as cited in Cunningham and Roberts, 2010). In simpler language, the visual image of a concept may not reflect the true structure of what is to be defined. Again, I feel that it is important to do hands on construction of geometric solids and in the process, to examine concepts like the height of an equilateral triangle, the diagonal of a pentagon, congruency and similarity.

The hands on analysis of geometric solids not only gives students a firm knowledge of the classical Platonic solids but helps to develop critical thinking and analysis skills which can be

translated to other disciplines, The ability to compare and contrast a particular geometric solids for example, certainly has its corollary in critical writing of any type.

Besides the pedagogical significance of this project there is also the benefit of creativity and play which the students experienced. Play as learning is very important in any academic discipline. My students were able to find great joy in constructing formal objects as well as those of their own design. Next semester, I am going to encourage that more and expect to see some creativity and spontaneity in their designs.

Beyond the project

It is my hope to continue working with Mckinnon Elementary School in developing a geometry lab which could be used by all grades: K-6. Susan Fisher, the principal and I have been discussing this as we both see the applicability of hands on instruction in geometry to all ages. There is a firm possibility that I will present the work of my students to the local school board to pursue the possibility of extending the geometry lab which I am establishing at Mckinnon Elementary school to other schools in the area. I also plan to refine what we have already developed.

Assessment

In order to assess the progress of my students I use several techniques each of which addresses a different aspect of their geometric education. The use of daily writing prompts, journals, periodic quizzes as well as one on one assessment with the students gives me an indication of their progress. The daily writing prompts which I use are adapted from the K-W-L method of Donna Ogle. They require the student to indicate what they know, what they want to know and what they want to learn (Ogle,1986) . I use the second two parameters of the method and incorporate the first into the notebook which I am having the students fill out as well. Dr. Paoze Thao of CSUMB uses these writing prompts in his linguistics, global education, and Asian history classes to apprise him of his student's daily progress and it is to him that I owe the idea for incorporating the same prompts in my Zometool workshop.

The notebooks are for the student's benefit but also a way for me to check that they are writing down their explanations of the formulas which we use in class as well as to do any drawings that would help them understand the structure of the solid which they are constructing. I have to really encourage them to write things down because they get so excited about building things that sometimes they lose track of the concepts which allow the building.

So far I have designed two quizzes to help assess the student's knowledge of geometry. Depending on the time that I have left to work with the students, I am planning on developing more as we discuss other formulas and concepts related to geometry. The first quiz was just on prefixes and interior angles of polygons. The second was to test their ability with straight edge and compass to do things like bisecting a line and inscribing various regular polygons in a circle.

I also do one on one assessment with the children. I find that by spacing what I have taught and repeatedly reiterating concepts like Euler's formula that the students are able to

gradually assimilate the concepts better than if I just mention them one day and move on to something new.

As my project continues to move forward I will undoubtedly incorporate more quizzes and seek for more focused responses from the students in both their writing prompts and notebooks. I am pleased so far with the progress of my students. I have included the writing prompt template as well as the two quizzes which I have designed in the appendices at the end of this paper.

Results

As stated in my literature review, I have been very interested in observing both the development of spatial structuring and an evaluation of how the Van Hiele Levels of my students would develop as time went on. Through direct observation, daily writing prompts and journals as well as the occasional quiz, I was able to see how my students were progressing.

The Van Hiele levels essentially are a formal way of describing how students are able to both identify shapes and also to see the relationship between composite parts. These levels capture the general trend in students' geometry thinking of moving from seeing a whole shape to looking at its parts and from basing their thinking on visual images to considering explicitly defined properties" (Ambrose & Kenehan, 2009,p.159).

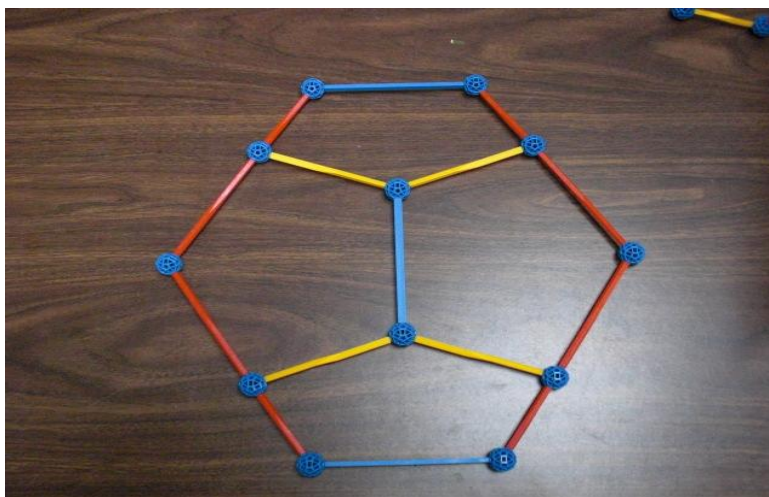
For the most part my work with the six 5th graders achieved the purpose of increasing their geometric perception and knowledge. I was not however, able to see a clear and discrete expression of a specific Van Hiele Level throughout my work with the students. Like Piaget's stages of cognitive development, the Van Hiele Levels are in fact a guide for observation and not an 'engraved in stone' way of evaluating the precise geometric ability of a student.

The first quiz which I gave the students was fairly successful with an average score of 90+. I was able to go over the quiz in class and make sure that all of my students were able to recite and understand both the prefixes and suffixes attached to 2 and 3 dimensional objects. The formulas which I quizzed them on were new and will require reinforcement next semester.

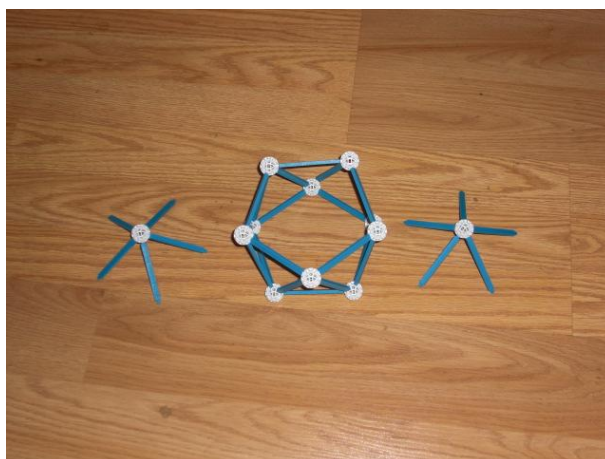
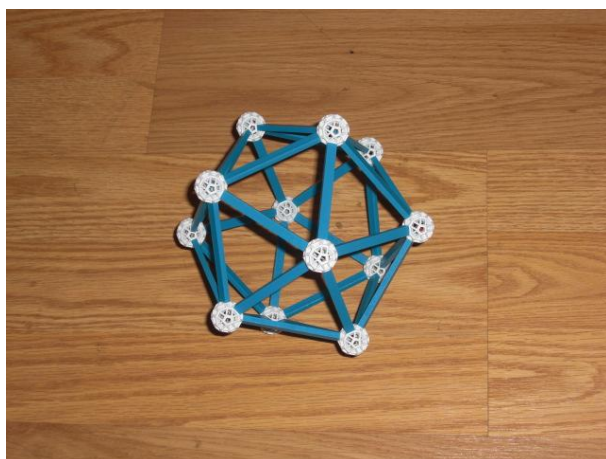
In a response to some questions which I had posed to David Henderson, a geometer at Cornell University concerning the Van Hiele levels, an email to me indicated that he did not accept the formal levels which the Van Hieles espoused. There are several levels which a student may be on at the same time and an upper level (say formal abstract thought) may express itself unexpectedly (Henderson, D., 2010). One of my students had problems with "seeing the trees because of the forest" when constructing a geometric object and would just put struts where he thought they would go rather than looking at the whole. However, he had great insights into 4 dimensional objects when we built the hyper dodecahedron.

Each student however had unique ways of constructing specific geometric cells or objects. I saw here an analogy between the choice of an algorithm used to solve arithmetic and the procedure used to construct a geometric figure or solid. In both cases the student uses the

method that makes sense to him or her. For example, the following flat cell of the hyper-dodecahedron may be constructed in several ways:



One student may construct the border of red and blue struts and then add the winged blue strut in the center. Another may do just the opposite. A third way would be to construct the individual pentagons which comprise the cell and then just connect them to form the whole. Again, I found the Van Hiele levels to be fluid and really non-specific in reality. What I did stress was the need for really seeing the composite structure of a solid and not to be satisfied with one's first impression. For example, if one looks at the icosahedron, one's first impression is that of a solid composed of 20 equilateral triangles. When one actually builds one though and deconstructs it the following can occur:



We now have a pentagonal anti-prism (two hexagons rotated 54 degrees in respect to each other) which may be augmented by two five ray star constructions. It is this type of conceptualization that I stressed and was able some real progress in the students with whom I was working.

Evaluation

I suppose like anyone starting a new project, that I have had successes and ‘failures’.. I feel that the ‘failures’ however, were really learning experiences that have encouraged me to do things differently next semester. Because this workshop was my capstone project I wanted to get as much productivity as possible from my students during the few months which we were together. Consequently we built a lot of models and did not do as much computation and compass work that I would have liked to. That will be remedied next semester. The students which I have worked with so far will be mentoring 6 new students next semester. The constructions will still occur but math concepts such as the Pythagorean Theorem and possibly the quadratic equation will be introduced as well.

The second quiz which I designed was intended to evaluate the students’ knowledge of compass and straightedge constructions. I was unable to give that quiz primarily because of technical difficulties. The compasses which we had to work with were very primitive and the students had a hard time working with them. The school has purchased new compasses and we will continue that work next semester.

Also, at the time that I started the workshop I did not have a vertex template for the students to work with to facilitate the patterns necessary to produce the interior angles for the regular polygons. Next semester, I plan to have that template printed out with examples of vertex patterns for all of the Zometool constructible regular polygons.

The six students whom I worked with naturally polarized themselves into three groups. Two of my students really liked the labels of the solids such as “icosahedron” and “dodecahedron”. These two became the core of my work with the math of polyhedra and took to it like ducks to water. The two female students on the other hand really had an inclination towards shape and form. For these two, the emphasis was on Zonahedra (constructions whose faces are all parallelograms). My last two students seemed to really favor the abstract so they became my experts on 4 dimensional objects.

I must also mention that the fact that not only is Zometool a great pedagogical tool to teach elements of algebra and geometry, it is also highly effective in engaging students that might not be interested in math in general. One of my students shows up at least 15 minutes early every Tuesday morning just to get into the classroom and start to work. The parents of the students are also happy since they report a real enthusiasm from their children for the bi-weekly workshops and a general increase in self-confidence in general.

Through working with these students I learned a lot myself not only about physical geometry but also about pedagogy. I am going to really stress note taking and reciprocal teaching next semester. Through conversations with both students and teachers I have found that this is not a common practice in the K-6 classroom. The keeping of journals is very important for the education of children. It is a way for them to take ownership over their education rather than just sitting and receiving information verbally that they soon forget.

So much of our education is designed to make students productive members of society. In the process what Vygotsky refers to as the action/meaning ratio is stressed (Vygotsky, 1., 1978, p. 100-101). I think that room should be made for creativity and spontaneity so that that ratio is reversed to meaning/action.

Why do we so stress action and performance over meaning? It seems to me that while the educational system is getting fairly adept at producing functional citizens room needs to be made for creativity and personal intellectual and artistic satisfaction as well.

My work with the children was satisfying on many levels. I was able to put away my own elitism or knowledge and really seek to see joy and beauty in my students through their own work, seeing an increase in the confidence levels of both my female and male students as well as an anticipation of what is to come next semester.

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Rebecca Ambrose of University of California at U.C. Davis: Rebecca has done extensive work with polyhedra and young children. She gave me much insight as to why the study of Polyhedra is important for children.

George Hart: George is the author of *The Geometry of Zometool*. He and I communicated several times via email to help me with conceptual problems surrounding the mathematics of polyhedra.

Paul Hildebrandt: Co- owner of Zometool and collaborator with me.

Dr. Donald Pierce: Donald is a math professor at CSUMB. Dr. Pierce emphasized the preciseness of the mathematical language and how that is transferrable to other academic disciplines.

David Richter: David is a math professor at Western Michigan University with whom I communicated via email and phone. He is an avid Zometool user and has contributed much to the understanding of how to construct n- dimensional solids through the use of Zometool.

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Appendix A: Journal of Zometool Workshops

The purpose of these workshops was to develop a sense of spatial structuring in my students through the use of Zometool. Rather than approaching these sessions with a firm set of daily lesson plans, my intent was to logically follow a progression from two dimensional to four dimensional thinking regardless of how many sessions it took to develop.

The concept of spacing or repeated verbal iteration of mathematical concepts is very important both to reinforce the material being presented as well as to help the students retain new and unfamiliar concepts. I made a decision that I was going to follow a general plan of education for my students but return frequently to subjects that we had discussed at the beginning of the workshops rather than simply following a set of lesson plans. I first taught my students to construct plane figures such as the regular square, equilateral triangle, pentagon, hexagon, and decagon. We then moved onto the Platonic Solids which are constructed of identical regular polygons (same side length, same interior angles and the same number of figures meeting at each vertex). The Archimedean solids are composed of two or more different regular polygons and may have a different number meeting at each vertex.

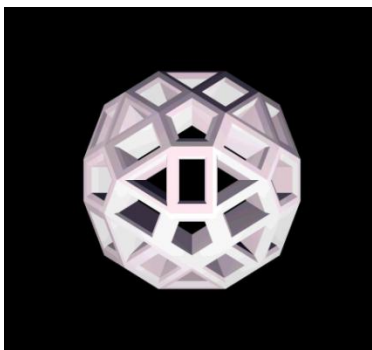
The development of 4 dimensional thinking takes a long time and very few mathematicians are really adept at it. For these workshops we built the 4 dimensional hypercube and hyperdodecahedron. Next semester we will be reviewing what we did this semester, introducing new concepts and also engage in reciprocal teaching and the monitoring of new students by the more experienced ones from this semester.

Zometool Workshop 1: 9/23/2010

The first session went fairly well. I explained the proper connection and disconnection procedures for inserting a strut into a Zometool ball as well as the correct method for removing it. (no twisting or bending to insert the struts and to remove the strut from the ball, use thumb and fingers carefully). We built the equilateral triangle and the regular pentagon, hexagon, and decagon.

I asked the students to make drawings in their notebooks as we went along. The students were asked to draw the vertex pattern of each polygon to help them construct it in the future. For example if one looks at the following picture of the Zometool connector:

One can see a vertically oriented rectangle flanked by two equilateral triangles. If two inserted into the rectangles at triangles, a 60 degree angle will be formed. By connecting the ends of the two struts, an equilateral triangle is formed from the 3 struts and three



oriented rectangle flanked by two equilateral triangles. If two inserted into the rectangles at triangles, a 60 degree angle will be formed. By connecting the ends of the two struts, an equilateral triangle is formed from the 3 struts and three

The history of the Zometool connector is interesting in that when the co-founders of Zometool, Paul Hildebrandt and Marc Pelletier approached several engineers to fabricate this heart of the Zometool system, nonone was willing to do it, saying that it was not possible. Undaunted, Hildebrandt and Pelletier learned the techniques of injection molding on their own and were able to perfectly reproduce the connector ball after one attempt.

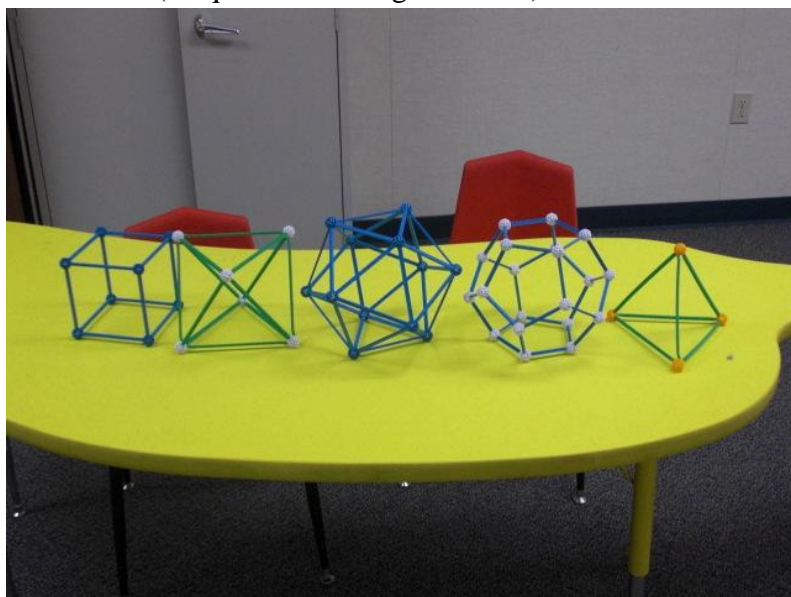
The same procedure was repeated for the other regular polygons using different vertex patterns. At the end of the session, I explained the concept of a plane which is defined by any two lines and how a polygon divides a plane into two areas: the inside of the polygon which is finite and the outside which is infinite.

One very interesting error occurred when I asked one of my students to construct a decagon. He understood that a decagon was composed of 10 struts and 10 balls. However when he put all of the components together, two of the line segments were actually in a straight line so in effect what he had was a nine-gon and not a decagon. I saw that I needed to show the class that a line segment contains an infinite amount of points and that Casey's two sides in the decagon were really just a long line segment composed of two shorter line segments connected by a point (the Zomeball connector).

I allowed 10 minutes for discussion, cleanup and the documentation of what they had learned in their folders and encouraged them all to attend the next session on 9/28/2010. During that session I plan to present more precise definitions of points, line segments, planes, and polygons. We also will be building the 5 Platonic solids: cube, tetrahedron, octahedron, dodecahedron, and icosohedron.

Zometool Workshops 2 and 3 (9/28 and 9/30)

This was a very good week for the students. We constructed the following Platonic Solids; The cube (six square faces), the octahedron (8 equilateral triangular faces), the icosahedron (faces composed of 20 equilateral triangles), the dodecahedron (12 pentagonal faces), and the tetrahedron (4 equilateral triangular faces).



This was a fun project for the students because they saw a wide variety of shapes from simple elements. For an additional exercise with these solids I had my students count the faces, vertices and edges of each solid. They were then to add the number of faces to the number of vertices and then subtract the number of edges for each solid. The answer is always 2 which is justified by Euler's formula $F+V-E=2$. They were

amazed by this and have committed the formula to memory.

One thing which I am stressing with the students is to see patterns both in the way we are constructing polyhedron as well as in any math calculations that we do in the process. When we were calculating the number of edges in a dodecahedron a student asked why since there five edges to a face and twelve faces that sixty was the number of edges. I commended her for her thinking but asked her how many faces shared an edge. When she saw that there were two faces to an edge she knew that she had to divide sixty by two to get thirty.

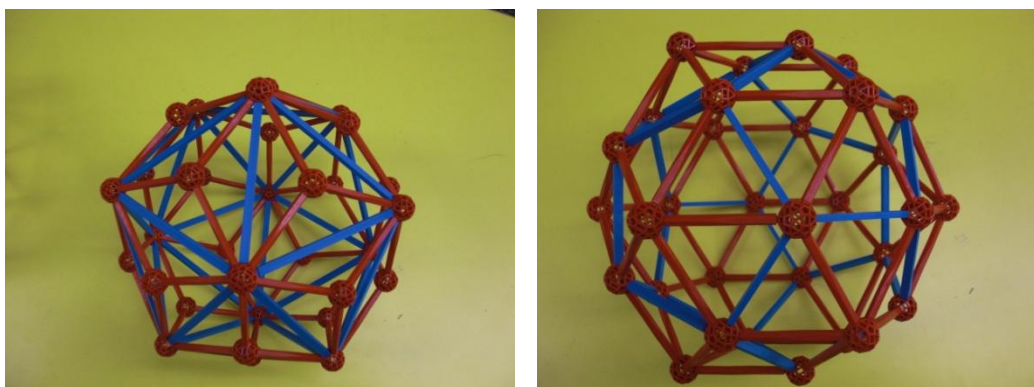
The obvious induction from this is that if one takes the number of edges two a face in a regular polyhedron, multiplies that number by the number of faces and then divides by two, one will get the number of edges for any polyhedron. The students are starting to think inductively and that is what I am after.

We also explored the concept of a dual where the vertices of one solid are at the midpoint of the face of another. For example the octahedron has 8 faces and six vertices. These vertices

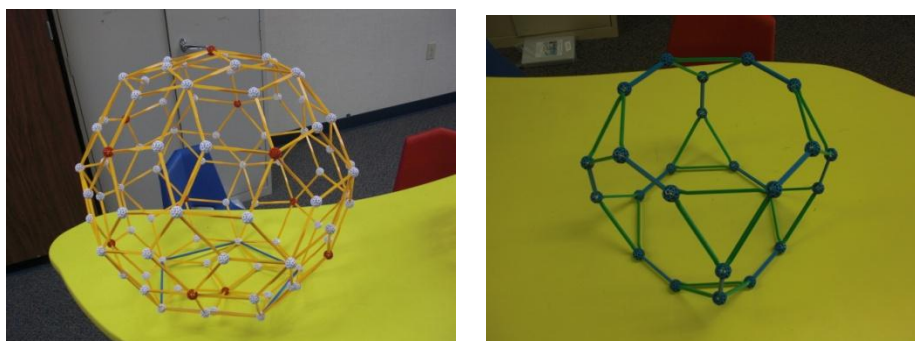
would coincide with the midpoints of the cube's 6 faces if the octahedron were inscribed in the cube. The reverse would also be true. The dodecahedron and icosahedron are duals of each other but the tetrahedron is only a dual of itself. Out of the six students that I have, there are a few that have not built the octahedron or tetrahedron yet. I plan to remedy that next week.

Zometool Workshops 3 & 4: 10/5/2010 & 10/7/2010

Tuesday we worked on some more polyhedra and also discussed how to find the interior angle of a regular pentagon using the formula $180 - 360/n$ where n equals the number of sides. Thursday we continued to work on various polyhedra such as the rhombic enneacontahedron and the rhombic triacontahedron. I had two of my students construct the red strut rhombic triacontahedron and also to inscribe both the dodecahedron and icosahedron in it to show the relationship between all three solids:



The two pictures below are the rhombic enneacontahedron and the truncated cube. The former consists of 90 rhombic faces (60 fat and 30 skinny). In order for the students to construct this, they had to see the regular dodecahedron as scaffolding. This structure is then easy to construct. To truncate the regular cube we 'slice' off a corner $1/3$ of the way along each edge to get the solid on the right.



Zometool Workshops 5 and 6: October 12 & October 14/ 2010

This was a very productive week for the students. On Tuesday, we went over the structure of the Platonic solids as well as how to write the Schafli code for them. For example, the cube consists of squares which meet three to a vertex hence $\{4,3\}$. The octahedron, on the other hand is composed of equilateral triangles which meet four to a vertex and its code is $\{3,4\}$ or the 'dual' of the cube. This relationship is called duality (the faces and vertices are reversed). The concept of duality was a new one for them. I am finding that I need to go over names and structure with my students frequently. They are learning though.

Yesterday, we worked on building the 120 cell hyper-dodecahedron which is the most complex model that the students will construct. The model itself consists of 5 sub structures which must be constructed carefully. All of the students did well though at times they could not 'see the trees because of the forest' and made some construction errors.

It was interesting to see this occur. What was happening was that the students at time could not discern the pattern in the sub cells and were just inserting struts where they thought that they should go. There are different cells to construct depending on whether one is expanding a face, vertex or edge and the students were confusing the three at times. This 'confusion' though was a learning experience for them though and we will finish the model this Thursday.

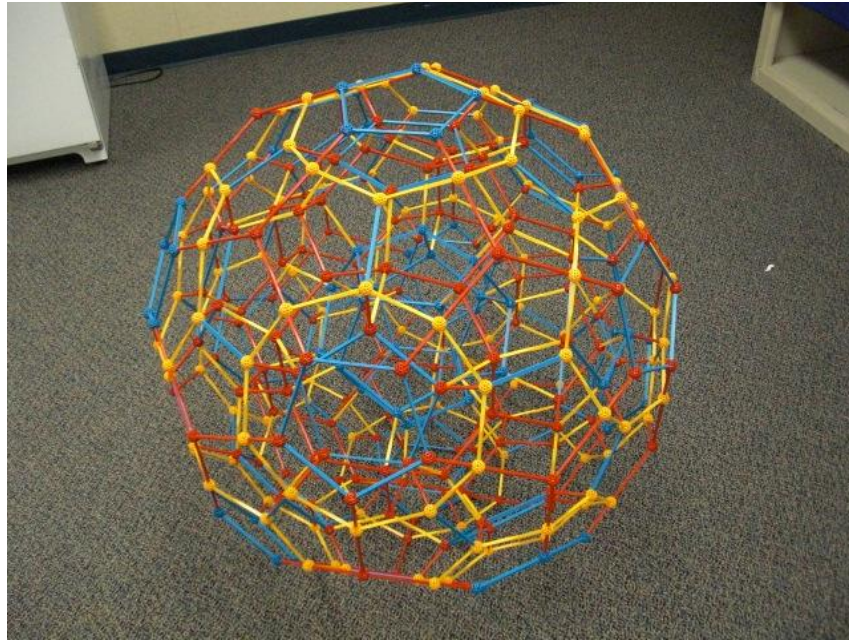
Zometool Workshops 6 & 7 (October, 19,21)

Tuesday we spent most of the morning doing compass and straight edge work. I had the student construct perpendicular bisectors of a given line and inscribe an equilateral triangle and hexagon into a circle. We will continue with that work shortly. I had planned to cover a lot more ground with the compass but will have to continue that during a later workshop. We finished construction of the 120 cell hyperdodecahedron as well. That was a challenge for the students because it required really using componential thinking as to which cells went where on the model.

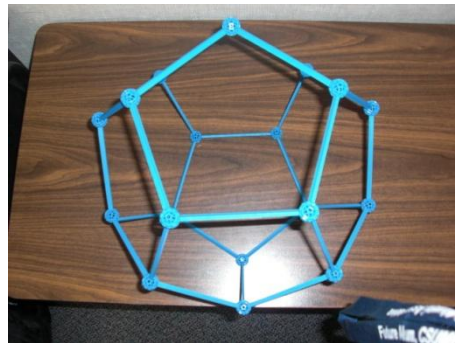
While developing spatialization skills with the children, I also want them to understand the mathematical concepts behind the construction and analysis of geometric solids. We went over Euler's formula again ($\text{Faces} + \text{Vertex} - \text{Edges} = 2$). In addition I introduced the Fibonacci sequence to the children. This sequence is important because the division of one Fibonacci number by the previous, tends to equal 1.618 which is the Golden Ratio.

We had finished the construction of the 120 hypercell the previous week but I wanted to go over the concept of the '4th' dimension with them again to make sure that they understood it. the end of the day I was happy and convinced that they had a working knowledge of the progression from the 0th to the 4th dimension.

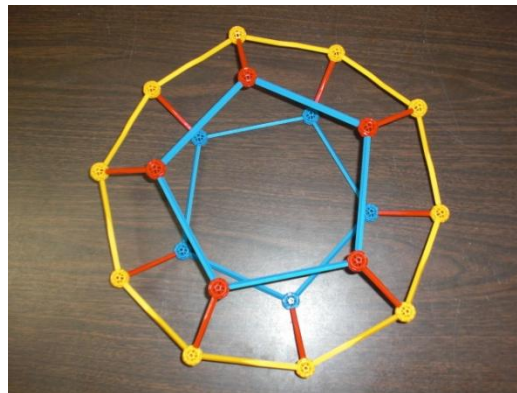
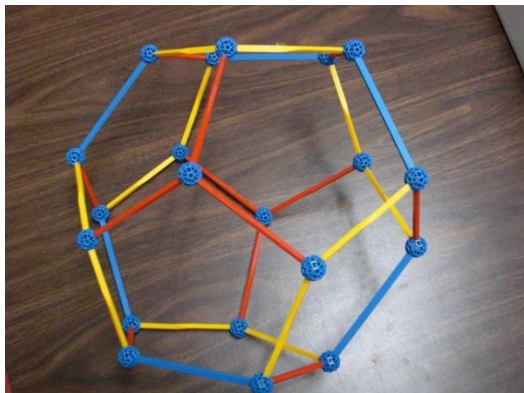
The following picture shows the completed 120 cell Hyper-dodecahedron:



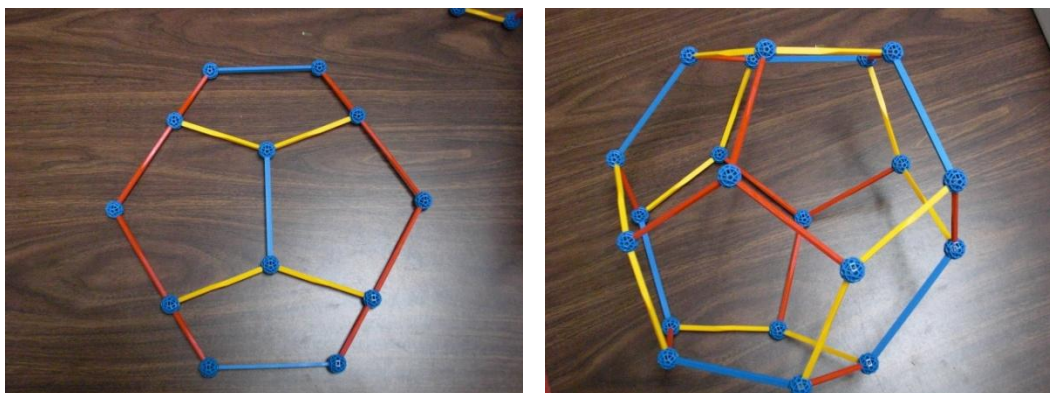
It is composed of the following: the core dodecahedron



Two squashed dodecahedrons for the faces:



Flat cell for the edges and skewed cell for the vertices:



With the construction of the 120 cell hyperdodecahedron, the formal part of my Zometool workshop has come to an end. During the next few weeks, the students and I deconstructed the polyhedra that we have already built and explored areas of geometry like tessellations and space-filling polyhedra. The workshops, in my mind were a success and I will continue the work that I have started with the students next semester.

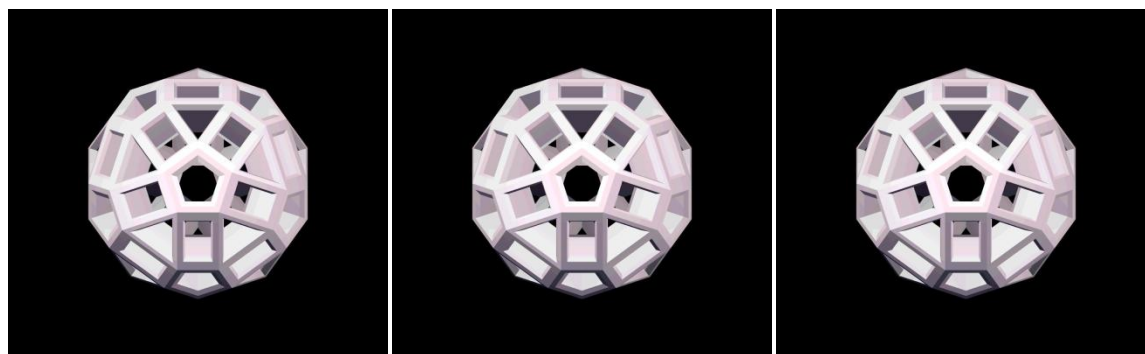
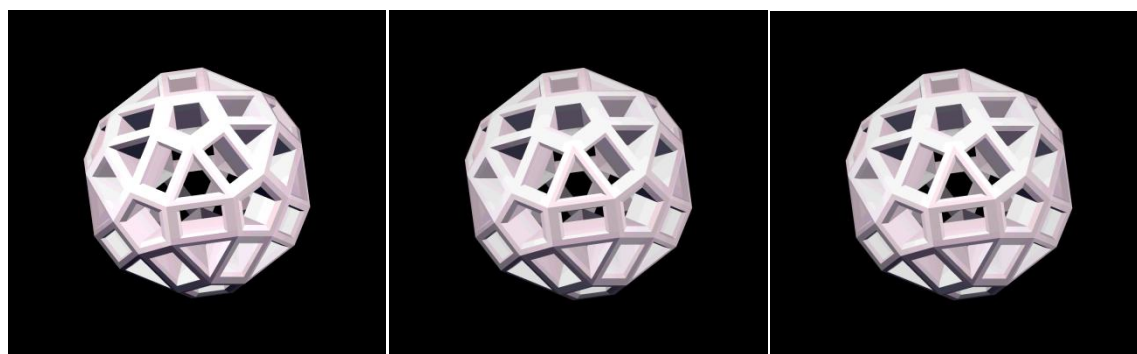
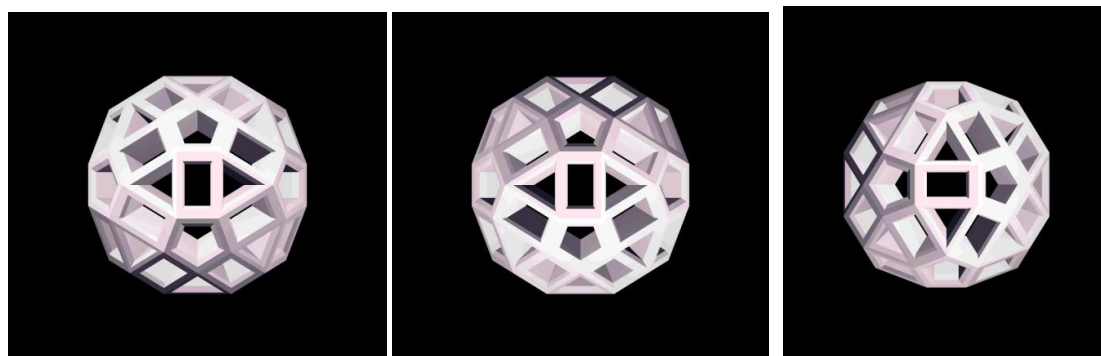
This semester's workshop encompassed many things. We learned about 2 dimensional polygons including the computation of their interior angles, how to define a plane using two lines, the composite relationship between different shapes and finally to initiate multidimensional thinking.

For my part I learned something about the different learning styles of my students as well as the dispelling of my conceptions about the Van Hiele levels as well as to get a greater appreciation of the beauty of mathematics and the 'birthing' of that beauty in my students. The following quote from Plato's *Symposium* sums of the esthetic of what I wanted to see as a result of my work:

For love, Socrates, is not,
as you imagine, the love of the beautiful only. '
'What then?' 'The love of generation and of
birth in beauty. '

It is my sincere hope that I can fulfill that quote as an educator. For if I can indeed 'birth beauty, I will consider myself a success. My work with the children was in one respect the developing of a resource. Also though, it was the building of relationships between the student and the teacher where the child strives towards being an adult and the teacher tries to recapture the joy of being a child learning new things. It was an interesting process. I saw growth and the development of mathematical thinking in my students and I had the joy of 'play'. This play for all concerned was not only play to learn but play as learning and for that I am thankful.

Appendix B: Zometool connector patterns
(Students should use colored pencils to color vertex patterns on templates)



Appendix C: Journal Entry or Writing Prompts

Name of Student: _____

Course: _____ Section _____

Semester: _____ Year _____

**PLEASE TAKE A FEW MINUTES AT THE END OF EACH CLASS SESSION TO COMMENT
ON “WHAT YOU HAVE LEARNED” TO THE INSTRUCTOR**

Week _____

Date: _____

Session 1: Student’s Comments

Date: _____

Session 2: Student’s Comments

Instructor’s Comments

Appendix D : Two quizzes

Quiz 1 : Common Prefixes and Interior angles

Name and Date _-----

Polygons (2 dimensional shapes) define prefix and give example

1: Tri =

2: Quadri =

3: Penta =

4: Hexa=

5: Hepta =

6: Octa =

7: Deca =

Polyhedrons (3 dimensional solids)

Tetra =

Octa =

Dodeca =

Triaconta =

Ennea=

Using the formula for the interior angles of a regular polygon $(180 - (360/n))$ where n = the number of sides give the interior angles for

1: Equilateral triangle

2: Square

3: Regular Pentagon

4: Regular Hexagon

5: Regular Octagon

6: Regular Decagon

Work carefully ☺

Quiz 2: Geometric constructions

Name and date.....

#1: Erect a perpendicular line at the spot marked x on the given line segment.

#2 Erect the perpendicular bisector to the given line segment.

3: Show how to bisect a given angle with a compass and straight edge.

#4 - # 8: construct in order an equilateral triangle, square, pentagon, hexagon and octagon within a circle.

Be sure to show your work carefully and indicate the interior angle and exterior angle of the polygons that you are inscribing.

#1

A-----x-----B

#2

A.....B

N.B: Pages 4-8 are blank in the original for the students' workspace

Appendix E: Glossary

P-gon (or Polygon) : A circuit of p line segments joining consecutive pairs of p points.(Coexeter, H. 1973, p.1) For example p in an equilateral triangle would be equal to 3

Polyhedron: A finite, connected set of plane polygons (Coexeter,H., p.4) . In other words a polyhedron is a three dimensional solid.

Common Prefixes:

Tri= 3

Quadri =4

Penta = 5

Hexa =6

Octa= 9

Deca = 10

Archimedean Solid: A solid in which there are two or more regular polygons which meet at each vertex. Each vertex is identical. For example the icosadodecahedron consists of 20 equilateral triangles and 12 regular pentagons meeting at each vertex.

Platonic Solids: A solid in which only one type of regular polygon meets at each vertex.

Cube: 6 squares- (3 meeting at each vertex)

Tetrahedron: 4 equilateral triangles- (3 meeting at each vertex)

Octahedron: 8 equilateral triangles (4 meeting at each vertex)

Dodecahedron: 12 regular pentagons (3 meeting at each vertex)

Icosahedron: 20 regular equilateral triangles (5 meeting at each vertex)

Euler's Formula: Faces + Vertices- Edges =2 (i.e. for a cube : 6 faces +8 vertices – 12 edges=2).

Descartes formula for Angular Deficiency: 360 degrees minus – the actual angle sum at each vertex. The sum of all of the vertices of a convex solid always equals 720 degrees. For example, the angular deficiency of a square is 90 degrees for each vertex.(90 degrees x 8 vertices equals 720 degrees total angular deficiency.)

Hypercube: Consists of 8 cubes and is a three dimensional projection of a 4 dimensional cube.

Hyper-dodecahedron: Consists of 120 dodecahedrons and is a 3 dimensional projection of a 4 dimensional dodecahedron.

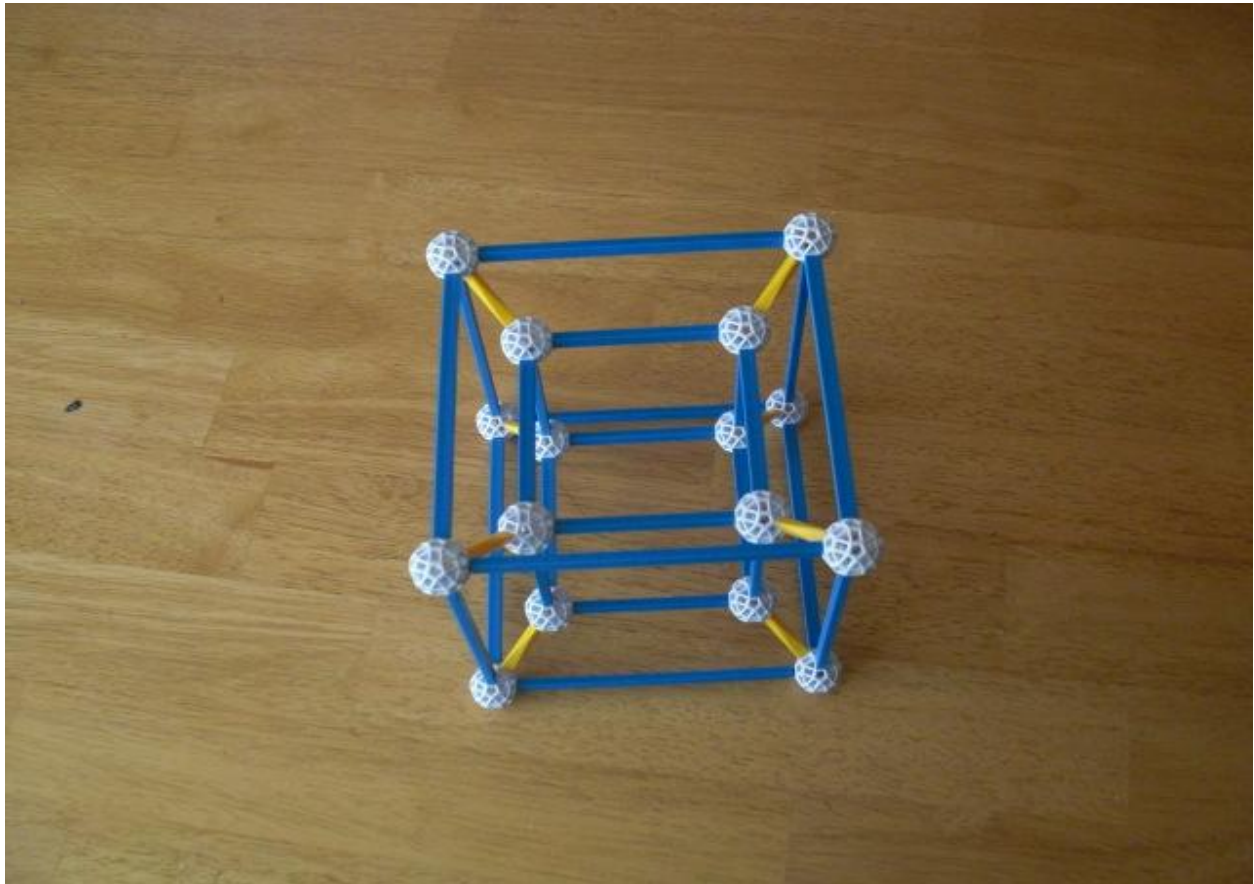
Tessellation: a tiling of n dimensional figures such as there are no gaps.

Appendix F: The Hypercube: A four dimensional cube

The photo below is an example of a 3 dimensional projection of a 4 dimensional object. The progression of thinking which goes into the construction of this object is as follows. Think of a point as being a 0 dimensional cube (The point = $2^0=1$ vertex). If we then extend this point perpendicularly we have a line segment = 1 dimensional cube with two vertices or end points ($2^1=2$ vertices). We can then project the line segment perpendicularly and we now have 4 points which we join into a square = 2 dimensional cube ($2^2=4$ vertices). We can now take the square and extend it perpendicularly from the 2nd dimension to the 3rd and we have a cube ($2^3 = 8$ vertices).

The final extension is the tricky one. Since we now have a three dimensional object, it is difficult to 'extend perpendicularly'. We therefore construct another smaller cube and put it 'inside' of the larger cube. This makes sense because we live in a 3 dimensional world and any projection of a 4 dimensional object is just a shadow projected into our world. So now we have 16 vertices all joined to make the hypercube ($2^4= 16$ vertices: one large cube, one small cube and 6 squashed cubes off of each face).

I



Appendix G: Useful websites

<http://georgehart.com/> (This is the home page for George Hart whose book *Zome Geometry* is very useful for anyone wanting to explore polyhedra and zometool constructions.

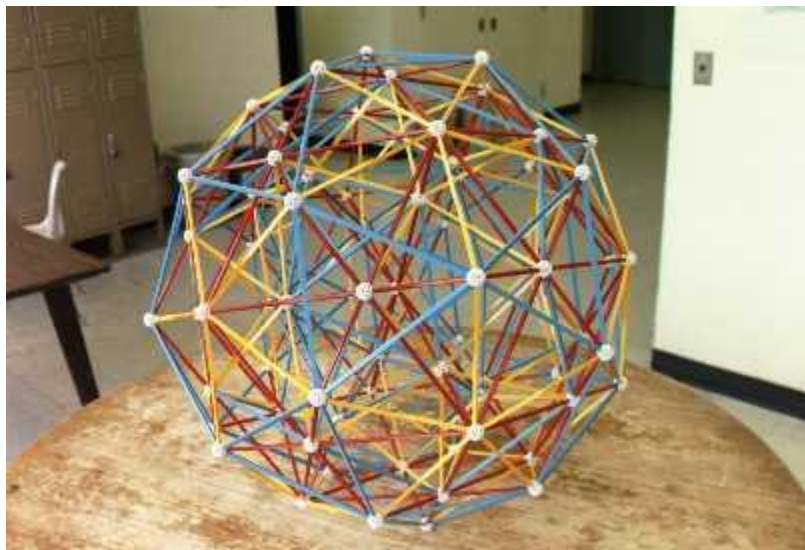
<http://homepages.wmich.edu/~drichter/zomeindex.htm> (David Richter is a math professor at Western Michigan University and a very avid Zomer,)

http://polyedergarten.de/e_info.htm (This is an incredible site for anyone wanting to learn more about polyhedra)

<http://www.wolfram.com/products/player/> (Wolfram offers this free player of Mathematica demonstrations. They also offer many free computer simulations of geometric constructions and the math which is involved in their structure.)

Appendix H: The Zometool Constructions of David Richter

The following images are used by permission of David Richter of Western Michigan University whose website (<http://homepages.wmich.edu/~drichter/zomeindex.htm>) shows some of the other possible constructions using Zometool.



This is the 600 cell which is the dual to the 120 cell hyperdodecahedron. It uses 75 balls, 72 R2 struts, 72 R3 struts, 120 B3 struts, and 120 Y3 struts. I plan to build this construction with my students next semester.

http://homepages.wmich.edu/~drichter/zome_600cell.htm



The first stellation of the 120 cell is a challenge and those wishing to construct this should first try the 120 cell mentioned earlier in this paper.

David's inventory for this construction is listed as:

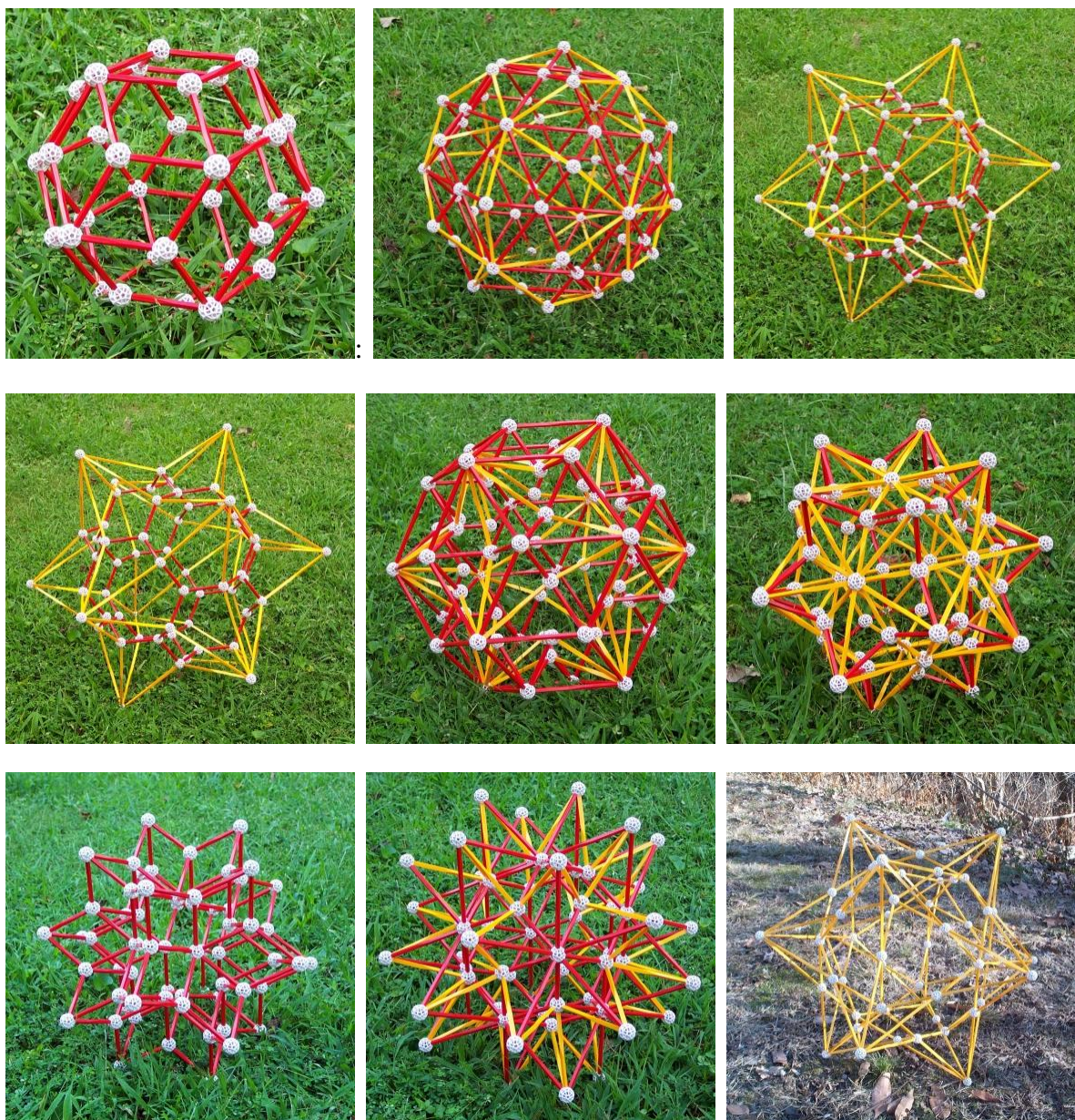
Balls: 385 Short Reds: 120 Medium Reds: 300
Long Reds: 180 Medium Yellows: 200 Long
Yellows: 400 Medium Blues: 180 Long Blues: 36

(<http://homepages.wmich.edu/~drichter/stellated120cell01.htm>)

The stellations of the rhombic triacontahedron

<http://homepages.wmich.edu/~drichter/triacontahedra.htm>

The rhombic triacontahedron is of particular interest to geometers because it contains within its structure the vertices of the icosahedron and dodecahedron. There are also many stellations possible with this figure. David Richter has shown a few of the possible thousand stellations in the following photos starting with the un-stellated version on the left. Part of the fascination of constructing polyhedral is seeing the diversity of shapes which may be generated from a simple construction such as the one on the far left in the first row of these photos.



Appendix I: Evaluation Letters

McKinnon Elementary School

2100 McKinnon Street, Salinas, CA 93906 (831) 443-7224

Susan Fisher, Principal

Fax (831) 443-7240

December 9, 2010

To Whom It May Concern:

Please accept this letter of recommendation and gratitude for Mr. Michael Read. Mr. Read has volunteered at McKinnon School this entire school year (2010-2011) of his own accord. Mr. Read was introduced to McKinnon School through his work at CSUMB. The CSUMB students of Dr. Marcia Karwas and now Dr. Andrea Kinney have provided service to these children for nine years.

In the fall of 2010 Mr. Read proposed an advanced geometry class for 6 students. These students received instruction every Tuesday morning from 7:45-9:00 and every Thursday afternoon from 2:40-4:00. Mr. Read held an open house for parents and all District administration in November.

The geometry class was so successful that Mr. Read offered to teach a remedial class for fifth graders still struggling with the multiplication tables. This class was offered from 12:20-1:30 on Wednesdays. The students love it and have grown immeasurably. For the remediation class 18 students were selected. The students loved the support. Mr. Read intends to continue the at-risk group in the spring semester. I believe he plans to use this semester's students as peer tutors.

Mr. Read has been a gift to McKinnon School. I believe that his experience with the children has provided him a hands on window view of the nature of this profession. The love of the children and the passion of "ah-ha" make teaching a great calling. I believe that Mike has this passion.

I commend CSUMB for the growing of excellent teachers. We have served as a student teaching site for many. Each one has been a joy to be around.

If you have any further questions concerning Mr. Read please do not hesitate to call.

Sincerely yours,


Susan Kay Fisher
(831) 443-7224

McKinnon Elementary School

2100 McKinnon Street, Salinas, CA 93906 (831) 443-7224

Susan Kay Fisher, Principal

Fax (831) 443-7240

December 4, 2010

To Whom It May Concern:

I have had the privilege of working with Mike Reed for the past three months. Mike worked with two groups of my students to strengthen and enhance their math experience.

Working with the advanced students, Mike introduced new concepts in his Geometry Workshop, enabling them to expand their thinking about how math works. His focus was to challenge the students in order to get them to apply their understanding of mathematical concepts in a new situation (building complex three dimensional structures), and to use mathematical reasoning as a way of finding answers to some of the problems that require more advanced knowledge.

In addition, Mike worked with a small group of my struggling students. These students are at-risk students for academic failure. Mike re-engaged them in "doing" math, by helping them to develop new strategies for enhancing their ability to perform basic computational skills, and applying those skills routinely and automatically.

My students eagerly looked forward to their math time with Mike, and came back to class with an enthusiasm and excitement that was truly wonderful! My students experienced a paradigm shift in their thinking and approach to math-- with my advanced students grasping the "bigger picture" of the structure and logic of math, and learning how to use the concepts effectively, while my struggling students began to develop a conceptual understanding of how and why math works.

Mike has been a wonderful addition to my math program, and has provided a great benefit to my students.

Sincerely,

Jyl Lutes

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